How Long is the Lebanese Coastline?
A Modern Answer to a Classic Question

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Received:	3 july 2025
Revised:	15 August 2025
Accepted:	4 September 2025
Published	15 September 2025

Abstract

Natural coastlines exhibit complex, irregular geometries as their length varies with scale. This study fills a gap by using Sentinel-2 satellite imagery and GIS techniques; NDVI was used for accurate surface land-water boundary extraction. Two methods were applied to quantify the coastline's fractal dimension. A stick (divider method) and a box length (box-counting method) from 10 to 20,480 meters was used. Within this range, the Lebanese coastline exhibits a fractal dimension of approximately 1.0805 using the box-counting method (95% CI: 1.072–1.088) and 1.0852 using the divider method (95% CI: 1.077–1.094), indicating low to moderate geometric complexity. The Mount Lebanon range runs parallel to the coast. The *D*-value of the Lebanese coastline is nearly identical to that of Turkey's Black Sea coast ($D \approx 1.08$), also controlled by a coast-parallel structure, but contrasts with the high complexity of Turkey's tectonic movement not parallel to the Aegean and Mediterranean shores ($D \approx 1.20$). This study applies multiple fractal methods to the Lebanese coastline to propose a standardized, multi-scale framework to resolve the 'coastline paradox'. We propose 'fit-for-purpose' Lebanese coastline lengths, ranging from 381.3km for fine-scale geomorphological studies to 215.6km for generalized national cartography. This range serves practical applications, including coastal zone management, infrastructure planning, sea-level-rise modeling, and the demarcation of maritime legal boundaries.

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Keywords: Fractal dimension, Box counting method, Divider method, Lebanese coastline length, GIS, NDVI, coastal geomorphology, Sentinel-2.

1. Introduction

Fractals are complex patterns exhibiting self-similarity or statistical self-similarity across various scales. The term, coined by Mandelbrot (1977), describes phenomena that are continuous but not differentiable, characterized by intricate detail at all levels of magnification. Fractal geometry studies these shapes, which, unlike standard Euclidean forms (lines, squares, spheres…), maintain their complexity under dilation or contraction (El-Shaarawi & Piegorsch, 2002).

A key descriptor in fractal geometry is the fractal dimension *(D)*, a non-integer value that provides a statistical index of a pattern's complexity and space-filling capacity (Mandelbrotet, 1983). A higher fractal dimension typically indicates a more convoluted or rougher pattern. This concept has found applications in diverse natural phenomena, including clouds, mountain topography, and, notably, coastlines (Mandelbrot, 1967).

The "coastline paradox," first highlighted by Richardson (1961), indicates that the measured length of a coastline depends on the scale of measurement; as the measuring unit decreases, the measured length tends to increase. This paradox is resolved by understanding coastlines as fractal-like objects, whose complexity is better captured by their fractal dimension rather than a single length value. Numerous studies have estimated the fractal dimension of various coastlines worldwide, with reported values typically ranging from D = 1.02 for relatively smooth coastlines (*e.g.*, parts of the Gulf of California) to D = 1.46 for highly intricate ones (*e.g.*, Delaware Bay, New Jersey) (Richardson, 1961; Mandelbrot, 1967). These studies have employed methods like the divider (ruler or stick) method or the box-counting method.

Furthermore, fractal geometry has been applied to other significant boundaries in the Eastern Mediterranean and the Arabian Peninsula, including the coastlines of Turkey (Yılmazer *et al*., 2021) and the land border of Saudi Arabia (Sajid *et al*., 2023). The land border of Saudi Arabia, analyzed using the box-counting method, is tectonically less complex, with D = 1.0878 (Sajid *et al*., 2023). While the more tectonically complex Mediterranean and Aegean shores of Turkey exhibit higher values of D (1.21 and 1.20 respectively), the Turkish Black Sea coast is less complex, with D = 1.08 (Yılmazer *et al*., 2021).

Previous estimates of the total length of the Lebanese coastline vary (*e.g.*, 200km to 225km according to Walley, 2001; El-Fadel *et al.*, 2010; The World Factbook, 2021), a systematic investigation of its fractal dimension, which would provide a scale-independent measure of its complexity, has not been previously undertaken. The Lebanese coastline, situated in the Eastern Mediterranean, is characterized by a mixture of rocky headlands, sandy beaches, river deltas, and significant urban development, particularly around major cities like Beirut, Tripoli, Saida, and Tyre (Walley, 2001; El-

Fadel *et al.,* 2010). This complex morphology presents challenges for consistent coastline delineation. Furthermore, artificial structures such as ports, marinas, and breakwaters significantly alter the natural coastline (Luijendijk *et al.,* 2018).

This study addresses a gap in research, on the Lebanese and eastern Mediterranean coastlines, by determining the fractal dimension of the Lebanese coastline using two established methodologies: the box-counting method and the divider (stick) method (*e.g.,* Mandelbrot, 1977; Feder, 1988; Turcotte, 1997). The primary contribution is the integration of Normalized Difference Vegetation Index (NDVI) derived from Sentinel-2 satellite imagery for robust and semi-automated extraction of the land-water interface (Jensen, 2007), followed by GIS-based fractal analysis algorithms (Burrough, 2015).

A key aspect of our approach is the inclusion of major artificial coastal structures to approximate the "actual" fractal dimension of the contemporary coastline (see *e.g.*, Burrough, 2015). We hypothesize that the Lebanese coastline exhibits fractal characteristics over a significant range of scales and that our methods can provide reliable and comparable estimates of its complexity.

2. Material and methods

Satellite Data and Coastline Extraction

The Lebanese coastline was extracted from Sentinel-2 MSI (MultiSpectral Instrument) Level-1C satellite imagery, acquired on June 11, 2024, to ensure a clear distinction between water and land surfaces. The imagery was selected based on minimal cloud cover (≤ 2%) observed on the acquisition date. Sentinel-2 provides multispectral data at spatial resolutions of 10m, 20m, and 60m; its use is supported by the Sentinel-2 User Handbook (European Space Agency [ESA], 2015). For this study, the 10m resolution bands, specifically Band 4 (Red, 665 nm) and Band 8 (Near-Infrared – NIR, 842 nm), were utilized due to their effectiveness in delineating coastal boundaries.

The Normalized Difference Vegetation Index (NDVI) was calculated to differentiate land from water bodies using the standard formula:

$$NDVI = \frac{NIR - RED}{NIR + RED}$$

Background on remote sensing and NDVI usage is provided by Jensen (2005) and Lillesand *et al.,* (2015). Pixels with NDVI values [*e.g.,* < 0] were classified as water, while pixels with NDVI values [*e.g.,* > 0] were classified as land. This threshold was determined after visual inspection and histogram analysis of NDVI values across the study area to ensure optimal separation. The resulting binary raster (land/water) was then converted into a vector polygon format using GIS software (ArcGIS®10.8). The land polygon representing Lebanon was selected, and its seaward boundary was extracted to define the coastline vector line. This vector line served as the input for both fractal analysis methods. The starting point of the coastline is located in the far south of Lebanon at coordinates *X*: 696325, *Y*: 3663704, and the ending point is located in the far north at coordinates *X*: 772781, *Y*: 3836433.

Inclusion of Artificial Features in Estimating Fractal Dimension

A pivotal aspect of this study was the comprehensive consideration of all elements along the Lebanese coast, including artificial structures, in our estimation of the coastline's fractal dimension. These structures, such as extensive port facilities (*e.g.,* Beirut, Tripoli, Saida, Tyre), breakwaters, and the Beirut-Rafic Hariri International Airport runway extending into the sea, were meticulously identified through a visual assessment of Sentinel-2 imagery, bolstered by higher-resolution base maps (*e.g.,* Google Earth, OpenStreetMap), and preexisting geographic knowledge.

Rather than circumventing these features, the coastline vector line was deliberately incorporated to include them, thus acknowledging their integral part in the contemporary coastal landscape. The justification for the inclusion of anthropogenic features in natural landscape analysis is supported by Rodríguez-Iturbe & Rinaldo, (1997). These man-made constructs are not mere appendages to the natural coastlines but are integral parts of the coastal environment that influence the physical, ecological, and socio-economic dynamics of the region. By retaining these features, we ensure a more comprehensive and realistic assessment of the coastline's complexity. This approach allows us to capture the full extent of the human footprint on the natural landscape, thereby providing a more nuanced understanding of the coastline's fractal dimension.

Box-Counting Method

The fractal dimension of the extracted and edited Lebanese coastline was estimated using the box-counting method. Falconer (2013) and Peitgen *et al.,* (2004) provide the theoretical basis for fractal geometry and the box-counting method. This method involves covering the coastline with a grid of square boxes of side length ε and counting the number of boxes $N(\varepsilon)$ that intersect the coastline. This process is repeated for a range of box sizes.

For a fractal object, $N(\varepsilon)$ scales with ε according to the power law: $N(\varepsilon) \propto \varepsilon^{-D}$

Where D is the box-counting fractal dimension. By applying logarithms, the relationship becomes linear: $Log_{10}(N(\varepsilon)) = -D \log_{10}(\varepsilon) + C$

Where C is a constant. Thus, D can be estimated as the negative of the slope of a straight line fitted to the points on a log-log plot of $N(\varepsilon)$ versus ε. $D = -m$

In this study, a series of 15 box side sizes (ε) were used, ranging from 10 meters (corresponding to the nominal resolution of the input Sentinel-2 data used for NDVI) and progressively doubling up to

(163.84km).For each box size ε, a fishnet grid was generated in ArcGIS® 10.8, and the number of boxes $N(\varepsilon)$ intersecting the coastline vector was counted. Base-10 logarithms were used for the transformation: $log_{10}(N(\varepsilon))$ versus $log_{10}(\varepsilon)$. Linear regression was then applied to these log-transformed data points to determine the slope, and D was calculated as the negative of this slope.

Divider (Stick) Method

The divider method, also known as the ruler or stick method, was also employed manually to estimate the fractal dimension. This method involves measuring the coastline's length $L(s)$ using a series of "sticks", or dividers, of a fixed length s (Mandelbrot, 1967). The coastline is traversed by $N(s)$ segments of length s, such that: $L(s) = N(s) * s$

This process is repeated for various stick lengths. For a fractal coastline, the measured length $L(s)$ scales with the stick length s according to the power law: $L(s) \propto s^{1-D}$

Where D is the fractal dimension. By Taking logarithms, this relationship becomes: $\text{Log}_{10}(L(s)) = (1-D)\log_{10}(s) + C'$

Where C' is a constant. Thus, D can be estimated from the slope (m) of a straight line fitted to the points on a log-log plot of $L(s)$ versus s: $D = 1 - m$

In this study, a series of 15 stick lengths (s) were used, ranging from 10m and progressively doubling up to 163,840m. For each stick length s, the coastline vector was segmented, and the total length $L(s)$ was calculated. Base-10 logarithms were used for the transformation: $\log_{10}(L(s))$ versus $\log_{10}(s)$. Linear regression was then applied to these log-transformed data points to determine the slope m, and D was calculated as 1 - m.

Key Criteria for the Optimal Scaling Regime

For calculating the fractal dimension of the Lebanese coastline:

1. Sufficient Number of Elements (N): The number of sticks (N) or boxes $(N(\varepsilon))$ should be large enough (*e.g.,* $N > 10$, preferably $N > 20$).
2. Consistency: The range should ideally be one where the fractal dimension is expected to be relatively stable for both methods.
3. Linearity in Log-Log Plot: The log-log plots for both datasets should be reasonably linear over this common range.

Table 1 presents the number of sticks (N) and boxes $(N(\varepsilon))$ required to measure the coastline at varying scales (s or ε). The comments indicate the reliability of data points based on N:

1. Excellent/Good Reliability: $N \geq 1000$ to $N \geq 10$, supporting robust fractal analysis.

2. Conservative Regime (Upper Limit): Scales up to 10,240m ($N \geq 20$), ensuring high statistical confidence.
3. Inclusive Regime (Borderline Threshold): Scales up to 20,480m ($N \geq 10$), balancing data retention and reliability.
4. Unreliable Regime: Scales ≥ 40,960m ($N \leq 6$), excluded due to finite-size effects and noise.

The table highlights the trade-offs between maximizing scaling range and maintaining data quality, guiding the selection of optimal regimes for fractal dimension calculation.

Table 1: Stick vs. Box method data for the Lebanese coastline's fractal dimension

Scale (m)	Stick Method: $N(s)$	Box Method: $N(\varepsilon)$	Comments
10	38127	43754	Excellent N for both.
20	17528	21709	Excellent N for both.
40	8058	10261	Excellent N for both.
80	3705	4745	Excellent N for both.
160	1703	2213	High N for both, good reliability.
320	775	1021	Good N for both.
640	362	496	Good N for both.
1280	174	237	Good N for both.
2560	85	113	Good N for both.
5120	41	55	Good N for both.
10240	20	24	N ≥ 20 for both. This is a very good upper limit for a conservative, highly reliable regime.
20480	10	12	N ≥ 10 for both. This is often considered a borderline but acceptable minimum for an inclusive regime.
40960	5	6	N is too small for both. Results likely unreliable.
81920	3	3	N is far too small for both. Results unreliable.
163840	2	2	N is far too small for both. Results unreliable.

Statistical Validation of the Fractal Dimension: Standard Error and Confidence Interval

The precision of the estimated *D* depends on the uncertainty associated with the slope of the regression line used in the log-log analysis of coastline data. The standard error of the slope, *SE*(*b*), provides a measure of this variability and is calculated by dividing the standard error of the residuals to the square root of the sum of squared deviations of the independent variable (Montgomery *et al*., 2021):

$$SE(b) = \frac{S_e}{\sqrt{S_{xx}}}$$

Where S_e represents the standard error of the residuals and S_{xx} is the corrected sum of squares of the independent *x*. *SE*(*b*) can be expressed as:

$$SE(b) = \frac{\sqrt{\sum(y_i - \hat{y}_i)^2 / (n - 2)}}{\sqrt{\sum(x_i - \bar{x})^2}}$$

y_i are the logarithms of the measurements: $\log_{10}(N(\varepsilon))$ for the box-counting method (where $N(\varepsilon)$ is the number of boxes covering the coastline) or $\log_{10}(L(s))$ for the divider method (where $L(s)$ is the measured coastline length). The corresponding x_i values are the logarithms of the measurement scales: $\log_{10}(\varepsilon)$ for the box-counting method or $\log_{10}(s)$ for the divider method. The $\hat{y}_i$ are the predicted values from the regression line, $\bar{x}$ is the mean of x_i, and n is the number of observations (Draper & Smith, 1998).

This standard error quantifies the confidence in the estimated slope and, by extension, in the calculated fractal dimension ($D = 1 - m$ for the divider method or $D = - m$ for the box-counting method). To provide a statistical measure of reliability, $SE(b)$ is used to compute a confidence interval (CI) for D. For a 95% confidence level, the CI is defined as: $CI = D \pm (t^* \cdot SE(b))$

Where t^* is the critical value from the Student's t-distribution with $n-2$ degrees of freedom. This interval represents the range in which the true fractal dimension is expected to lie with 95% confidence, accounting for the variability in the slope estimate (Montgomery *et al*., 2021).

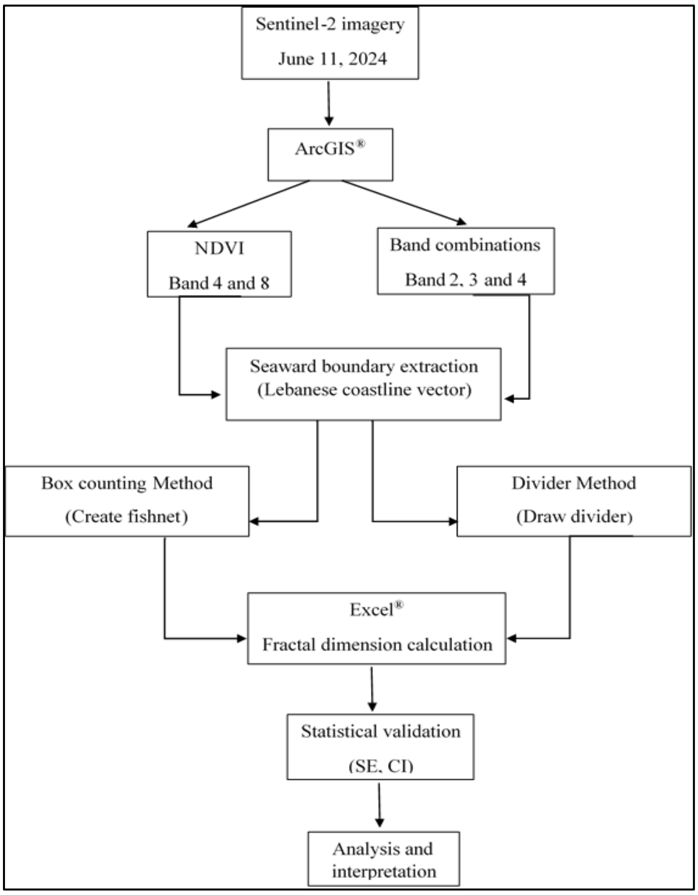

Fig 1. Flowchart of the adopted steps for calculating the Lebanese coastline's fractal dimension.

3. Results

Box-Counting Method

The application of the box-counting method to the derived Lebanese coastline yielded the data presented in Table 2. The smallest box size (ε = 10m) required 43,754 boxes to cover the coastline, while the largest box size (ε = 163,840m) required only 2 box. A log-log plot of $N(\varepsilon)$ versus ε was generated from these data (Fig. 2). Visual inspection of the plot shows a strong linear relationship across the range of scales tested.

Linear regression analysis performed on the $\log_{10}(N(\varepsilon))$ versus $\log_{10}(\varepsilon)$ data yielded a slope *(m)* of approximately -1.0805. The fractal dimension *D* is the negative of this slope:

$$D = -m = -(-1.0805) = 1.0805$$

The coefficient of determination (R^2) for this linear regression was approximately 0.999, indicating an excellent fit of the data to the power-law model and strongly supporting the fractal nature of the Lebanese coastline over the scales examined. Fig. 3 shows three examples, comparing fewer larger boxes (*e.g.*, box size 10240m, *N* = 24) with more numerous, smaller boxes (*e.g.*, box size 2560m, *N* = 113) needed to cover the coastline. The standard error expresses the confidence in the result. The standard error of the slope was calculated as

$$SE(b) = \pm\ 0.0037,\ \text{(rounded to } \pm\ 0.004\text{)}.$$

A 95% confidence interval (CI) was calculated as: $CI = D \pm (t^* \cdot SE(b))$

With $D = 1.0805$, $SE(b) = 0.0037$, and a critical $t^* = 2.262$ (for a 95% confidence). Substituting these values gives: $1.0805 \pm (2.262\times0.0037) = 1.0805 \pm 0.00837$

Thus, the 95% confidence interval is: $D \in [1.072, 1.088]$

This narrow interval demonstrates the robustness of the regression model and provides strong confidence in the estimated fractal dimension of the Lebanese coastline.

Table 2: Box-counting data for the Lebanese coastline.

Box Size ε (*m*)	Number of Boxes $N(\varepsilon)$	$\log_{10}(\varepsilon)$	$\log_{10}(N(\varepsilon))$
10	43754	1	4.641
20	21709	1.301	4.336
40	10261	1.602	4.011
80	4745	1.903	3.676
160	2213	2.204	3.345
320	1021	2.505	3.009
640	496	2.806	2.696
1280	237	3.107	2.375
2560	113	3.408	2.053
5120	55	3.709	1.74

10240	24	4.01	1.38
20480	12	4.311	1.079
40960	6	4.612	0.778
81920	3	4.913	0.477
163840	2	5.214	0.301

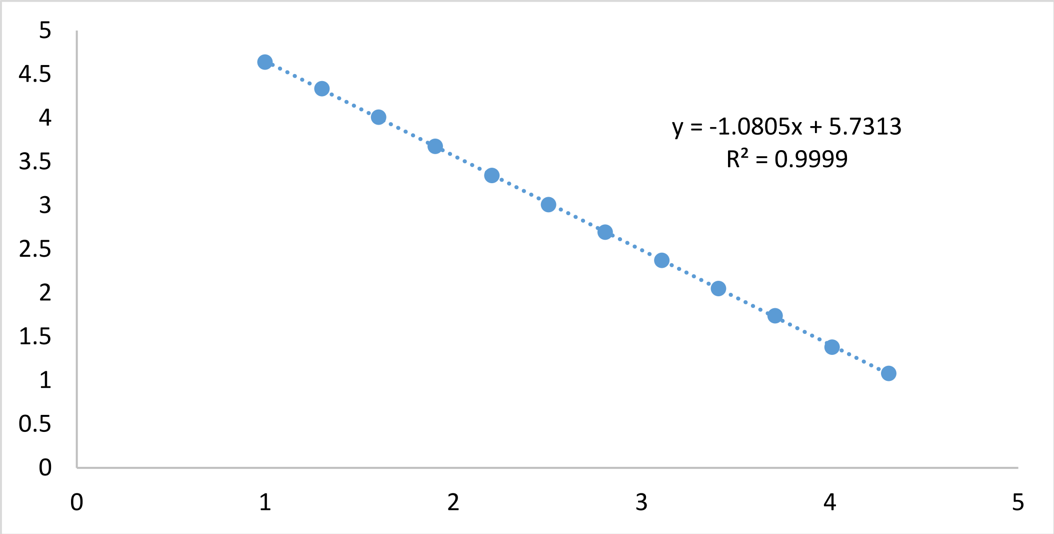

Fig. 2: Log-log plot of the number of boxes $N(\varepsilon)$ versus box side length ε for the Lebanese coastline (Box-Counting Method). D = 1.0805, R^2 = 0.999.

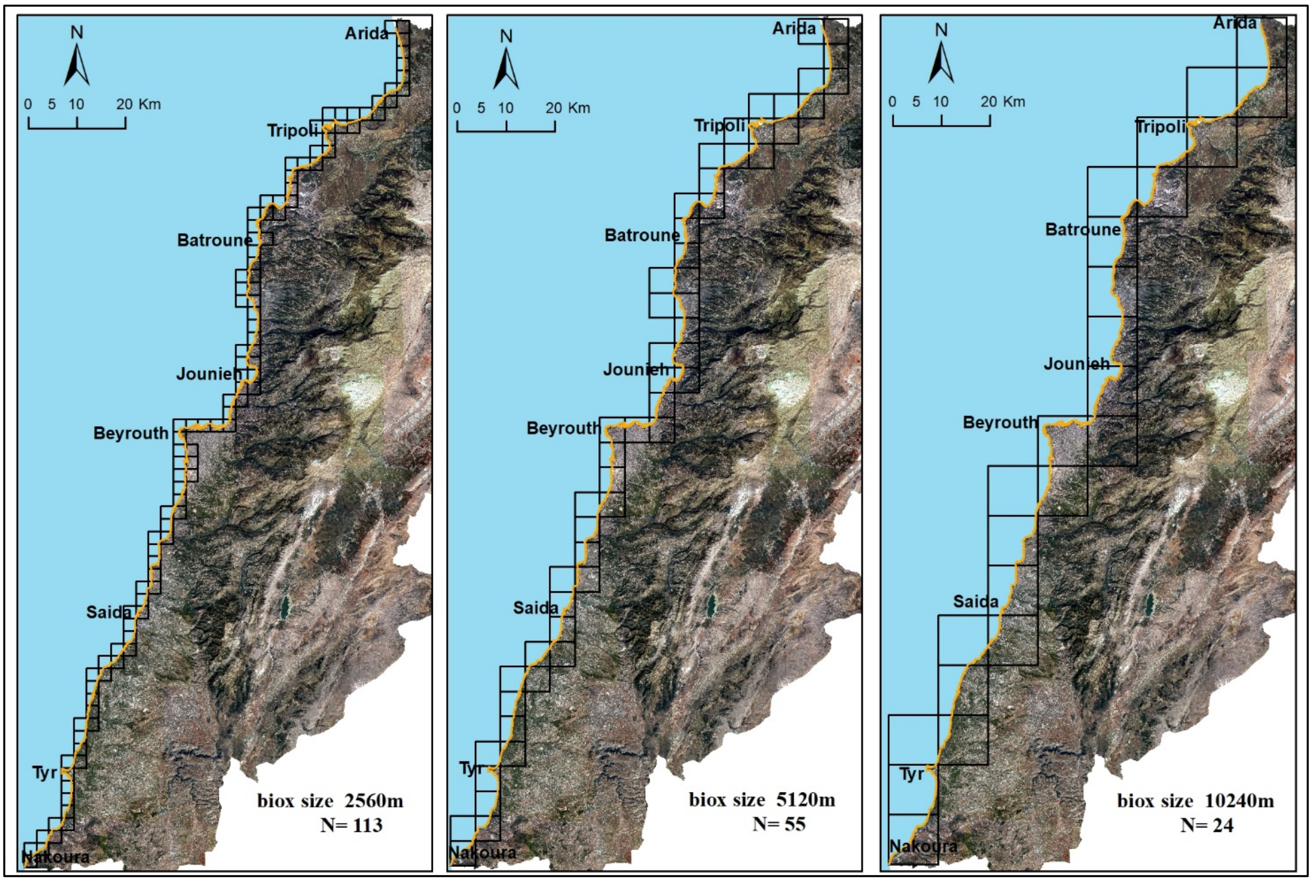

Fig 3. Illustration of the Box-counting method used to calculate fractal dimension of the Lebanese coastline with different box sizes.

Divider (Stick) Method

The application of the divider method yielded the data presented in Table 3. The measured coastline length $L(s)$ decreased from 381.270km for a stick length of 10m to 189.489km for a stick length of 163,840m. The application of the divider method yielded the data presented in Table 3. For the determination of the fractal dimension, the linear regression analysis focused on a specific range of stick lengths, specifically from 10m up to (but not including) 20,480m. This range was selected because it most consistently exhibited the linear power-law relationship in the log-log plot of $L(s)$ versus s, which is crucial for a robust D value and avoids the 'saturation effect' observed at larger stick lengths.

A log-log plot of $L(s)$ versus s was generated from these data (Fig. 4). Visual inspection shows a strong linear relationship for smaller stick lengths, with a flattening trend for larger stick lengths where the measured length approaches a constant value. For the regression analysis, typically points exhibiting this saturation are excluded.

Linear regression analysis performed on the $\log_{10}(L(s))$ versus $\log_{10}(s)$ data, for $s < 20480$, yielded a slope (m) of approximately -0.099. The fractal dimension D is calculated as:

$$D = 1 - m = 1 - (-1.0852) = 1.0852$$

The coefficient of determination (R^2) for this linear regression was approximately 0.999 indicating a strong fit of the data to the power-law model over the scales examined. The standard error of the slope was ± 0.0037 (rounded to ± 0.004), providing a measure of variability in the slope estimate.

A 95% confidence interval (CI) was computed as: $CI = D \pm (t^* \cdot SE(b))$

With $D = 1.0852$, $SE(b) = 0.0037$, and a critical $t^* = 2.262$. Substituting these values gives: 1.0852 ± (2.262×0.0037) = 1.0852 ± 0.00837

Thus, the 95% confidence interval is: $D \in [1.0768, 1.0936]$

Fig. 5 illustrates the divider method, showing how increasing the 'Stick length' (from 2,560m to 10,240m) reduces the number of segments (N from 85 to 20) and the total measured coastline length (from 215,573m to 197,512m).

Table 3: Divider (Stick) method data for the Lebanese coastline.

Stick Length s (m)	number of sticks $N(s)$	Length (m)	$\log_{10}(s)$	$\log_{10}(L(s))$
10	38,127	381,270	1	4.5812
20	17,528	350,560	1.301	4.2437
40	8,058	322,320	1.6021	3.9062
80	3,705	296,378	1.9031	3.5688
160	1,703	272,480	2.2041	3.2312
320	775	247,857	2.5051	2.8893
640	362	231,450	2.8062	2.5587

1,280	174	222,324	3.1072	2.2405
2,560	85	215,573	3.4082	1.9294
5,120	41	207,363	3.7093	1.6128
10,240	20	197,512	4.0103	1.301
20,480	10	196,233	4.3113	1
40,960	5	189,465	4.6123	0.699
81,920	3	189,505	4.9133	0.4771
163,840	2	189,489	5.2143	0.301

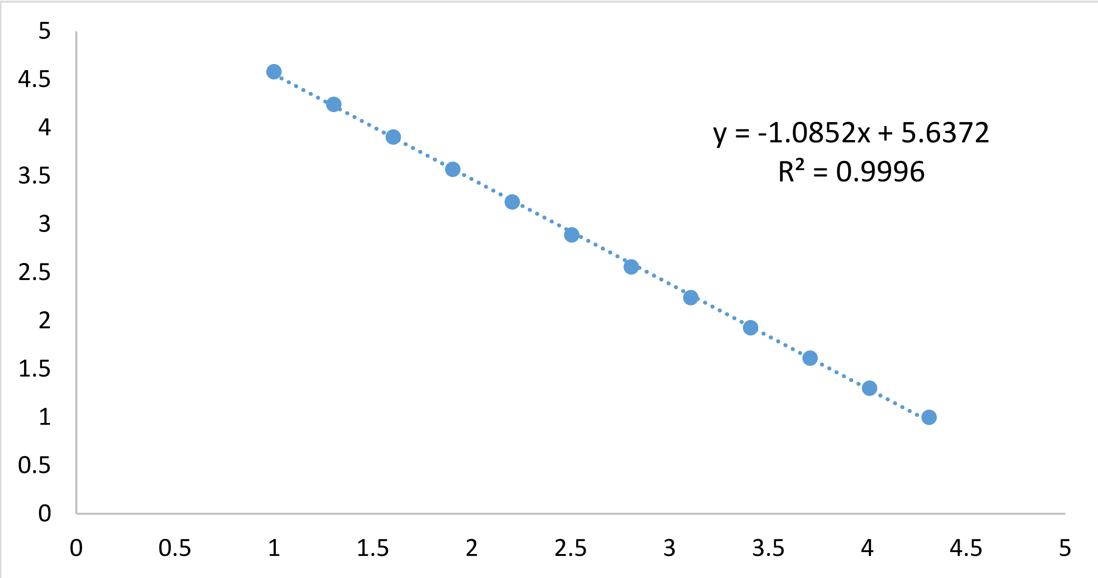

Fig. 4: Log-log plot of the measured coastline length $L(s)$ versus stick length s for the Lebanese coastline (Divider Method). $D = 1.0852$, $R^2 = 0.999$.

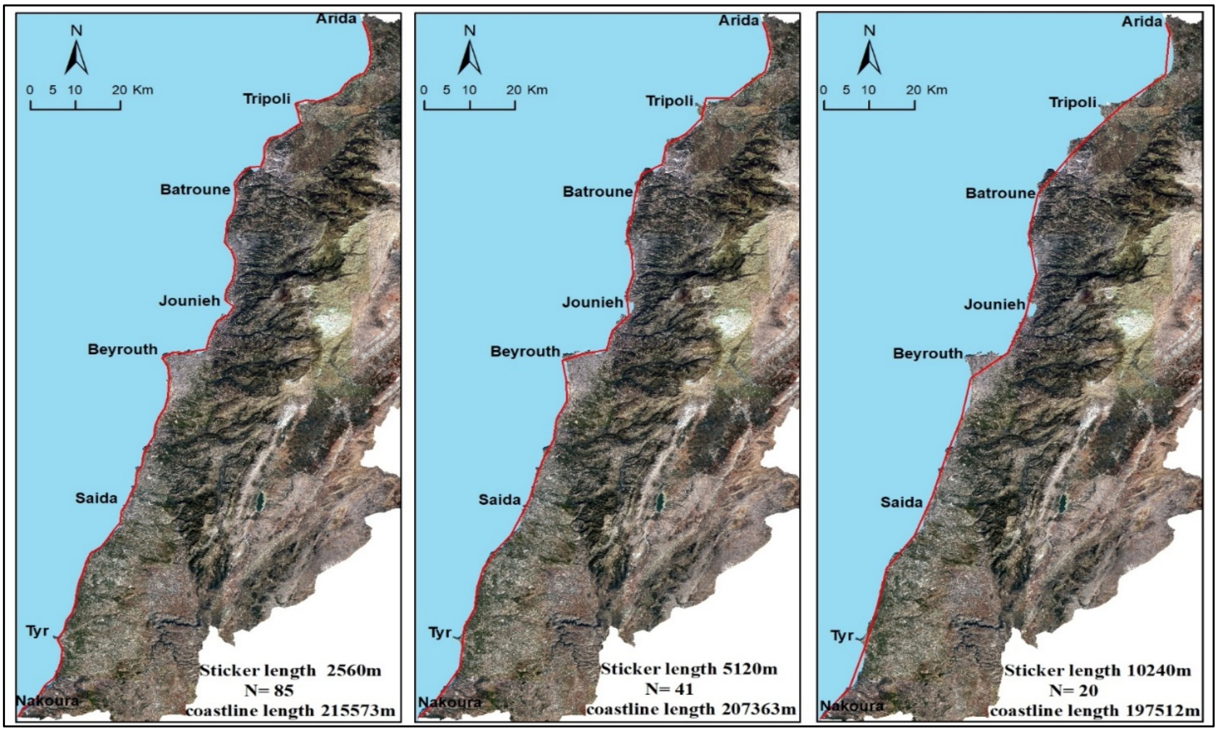

Fig 5. Illustration of the Divider (Stick) used method to estimate the Lebanese coastline length with different stick lengths.

4. Discussion

The Optimal Scaling Regime

1.Conservative & Highly Reliable:

- Scale from 10m to 10,240m.
- Within this range, the number of elements (N or $N(\varepsilon)$) is always ≥ 20.

2.Inclusive (and practically effective):

- Scale from 10m to 20,480m.
- Within this range, the number of elements (N or $N(\varepsilon)$) is always ≥ 10. The linear regressions using this range gave $D \approx 1.08$ with excellent R^2 values.

Performing linear regressions over this specific range (10m to 20,480m) yielded highly consistent fractal dimension values ($D = 1.0852$ for sticks, $D = 1.0805$ for boxes) and excellent R^2 values (0.999). This empirical evidence supports the robustness of this range.

Interpretation of the Fractal Dimension

A fractal dimension around 1.0805 to 1.0852 suggests that the Lebanese coastline is more complex and irregular than a simple Euclidean line ($D = 1.0$) but is relatively smooth compared to many highly indented or convoluted coastlines reported in the literature, which can have D values approaching 1.3 or higher (*e.g.*, Norway, fjords, intricate deltas) (Jiang *et al.*, 2020; Mandelbrot, 1982). The obtained values fall within the lower end of the typical range observed for natural coastlines globally (1.02 to 1.46). This indicates a coastline with a moderate degree of embayments, headlands, and irregularities at the scales resolved by the 10m imagery.

Despite the inclusion of major port structures and an airport runway, the calculated D of the coastline did not significantly increase. This stability can be attributed primarily to:

1. The limited cumulative length of artificial structures compared to the extensive and naturally complex shoreline, whose inherent ruggedness likely overshadows localized additions.
2. The dispersed, rather than concentrated, distribution of these artificial features, preventing a substantial elevation of the overall D value.

The natural coastline's irregularity appears to absorb the complexity from these structures, leading to a modest overall change in dimensionality.

Comparison of Fractal Dimension Estimates from Box-Counting and Divider Methods

The two methods yielded very similar fractal dimensions ($D_{box} = 1.0805$; $D_{divider} = 1.0852$). This congruence strengthens the overall conclusion about the low to moderate complexity of the Lebanese coastline. The slight difference ($\Delta D = 0.0047$) can be attributed to several factors:

1. Methodological Underpinnings: The box-counting method assesses how the number of boxes needed to cover the feature changes with box size, essentially measuring how the feature fills space.

The divider method measures how the apparent length of the feature changes with the length of the measuring unit. While both capture fractal scaling, their sensitivity to different aspects of geometric irregularity can vary.

2. Implementation Details: Minor differences in the precise implementation of algorithms (*e.g.*, how edge cases are handled for boxes, or how "walking" the divider is performed digitally) can lead to small variations.

Despite these minor differences, the close agreement between the two methods validates the robustness of the fractal characterization. Both confirm that the Lebanese coastline is only mildly irregular.

Structural Controls on Coastal Morphology and Fractal Dimension

The analysis of the Lebanese coastline reveals a relatively low fractal dimension (D), a characteristic that appears incongruous with the region's known complex tectonic setting and high density of mapped geological faults. However, a detailed examination of the main structural divisions of Lebanon provides a possible resolution to this apparent paradox.

Our findings suggest that the small value of the fractal dimension is primarily attributable to the distinct structural anisotropy of the Levant passive margin, where the dominant fault systems exhibit orientations largely parallel or sub-parallel to the coastline. Major structures, such as the Yammouneh and Serghaya faults, are significant transcurrent systems that predominantly traverse the hinterland, accommodating regional tectonic stresses but exerting limited direct influence on the fine-scale segmentation of the coastal boundary. While the Roum Fault lies closer to the coast, its general NNE-SSW trend also aligns with the coastal orientation.

Furthermore, a significant proportion of the numerous minor faults, often splaying from or subsidiary to these major systems, terminate before reaching the littoral zone. Consequently, they do not contribute extensively to the development of intricate coastal indentations across multiple scales, which would otherwise elevate the fractal dimension.

Although numerous minor faults originating from inland regions intersect the Lebanese coastline, inducing localized structural complexities, their scale and distribution are insufficient to significantly alter the overall fractal dimension value, owing to their subsidiary rather than major geological status.

The shoreline of Saudi Arabia along the Red Sea, encompassing the tectonically stable interior of the northward-drifting Arabian Plate, is an active continental rift margin, formed as the Arabian Plate separates from the African Plate. This ongoing extension has resulted in significant tectonic activity, causing pronounced crustal thinning, faulting, and the formation of a high-relief landscape characterized by the uplifted Red Sea Escarpment. This tectonic uplift drives high rates of erosion and incision, creating a morphologically complex coastal plain dissected by numerous wadi systems.

While the overall trend of the rift is linear on a continental scale, this active erosion and faulting creates a somehow rugged shoreline, which yields a fractal dimension of 1.0878 when analyzed with the box-counting method (Sajid *et al*., 2023).

This value is comparable to findings from the Turkish Black Sea coast, which, despite being in a tectonically active region,, presents a relatively linear shoreline governed by major structures such as the North Anatolian Fault Zone paralleling the coastline, and consequently also having a low *D* of 1.08 (Yılmazer *et al*., 2021). A similar low value is obtained in this study on the Lebanese coastline (*D* = 1.08), where also the Mount Lebanon range extends parallel to the Mediterranean Sea. In contrast, the highly indented and geometrically intricate Aegean and Mediterranean shores of Turkey yield significantly higher *D* (about 1.20), reflecting a deformation that is the result of the gravitational rollback of the subducting African Plate slab, a process that pulls apart the overriding Anatolian plate, creating the region's characteristic horst and graben structures.

The consistency of these findings across different locations in the region strengthens the interpretation of the fractal dimension as a useful indicator of the underlying geomorphological and tectonic controls on coastal geometry.

Methodological Considerations and Limitations

The methodologies employed, combining NDVI for coastline extraction with ArcGIS®10.8-based fractal analysis, proved effective. The use of Sentinel-2 imagery provides a balance between spatial resolution (10m for the bands used) and broad area coverage, suitable for national-scale assessment, not without limitations:

1. Resolution: The smallest box size (10m) and smallest stick length (10m) are dictated by the input imagery resolution. Finer details of the coastline below this scale are not captured, which could influence the *D* value if higher-resolution data were used. However, the consistency of the log-log plots across several orders of magnitude suggests robustness for the observed scales.
2. NDVI Thresholding: While NDVI is effective for land-water separation, the choice of threshold can introduce minor variations in the exact position of the delineated coastline, especially in areas with mixed pixels or turbid waters (though the latter is less of a persistent issue for the Lebanese coast).
3. Delineation of Artificial Structures: While this study deliberately included artificial structures to reflect the contemporary coastline, the manual editing process to precisely trace these complex features (*e.g.,* port outlines, breakwaters) around the land-water interface can introduce a degree of subjectivity. Alternative interpretations or minor variations in tracing the "shoreline" path in these highly modified areas are possible. A standardized protocol for delineating such features would be desirable for enhanced inter-study comparability.

4. Tidal State: The coastline position can vary with tidal state. The Sentinel-2 image represents a single snapshot in time. For coastlines with significant tidal ranges, this would be a more critical factor. The Mediterranean has a micro-tidal regime, so this effect is likely minimal for Lebanon.
5. Dynamic Nature: Coastlines are dynamic. The calculated D values are representative of the coastline's morphology at the time of image acquisition.

Fit-for-Purpose Lebanese Coastline Lengths

The coastline possesses no single, absolute "true" length. As demonstrated by the Divider (Stick) method data in Table 3, the measured length, $L(s)$, is intrinsically dependent on the scale of measurement, s. From a practical standpoint, the most useful scale for defining the coastline length should:

- Be consistent with international standards (*e.g.*, those used by national mapping agencies).
- Reflect the level of detail necessary for coastal management, planning, and environmental monitoring.
- Avoid arbitrary or excessively fine resolutions that may over-represent minor features or artificial structures.

The "Theoretically Justified Stick Length (ε)" for each discipline is derived from the principle of cartographic generalization. This involves aligning the measurement unit (ε) with the discipline's typical scale of observation and the real-world size of its features of interest. The selected ε thus acts as a calibrated spatial filter, ensuring it is fine enough to resolve the relevant coastal morphology for a given application, yet coarse enough to smooth over insignificant variations that would be considered cartographic noise at that specific analytical scale. The principle of matching the stick length to the "scale of the phenomenon" does not yield a single, unique value but rather a plausible *range* of values. The choice is arbitrary in the sense that other nearby values would also be justifiable; its strength lies in being a representative choice from within that rationally defined range.

Socio-Economic and Environmental Applications

- Urban Planning and Infrastructure: Using a high-resolution coastline length of 381.3km can be instrumental for municipal-level planning, helping to accurately define setbacks, regulate construction, and manage extensive port facilities.
- Environmental Protection and Marine Biodiversity: The Lebanese coast suffers from pollution, with untreated sewage and solid waste frequently dumped directly into the sea, severely degrading marine habitats. The suite of standardized lengths allows for more precise demarcation of protected areas, such as the Tyre and Palm Island nature reserves.

• Tourism and Economic Development: A standardized coastline length of 272.5km, suitable for topographic mapping, for minimizing environmental impact.

Climate Change Adaptation and Coastal Resilience

- Sea-Level Rise Modeling: The length of 350.6km (at a 20m stick length) is specifically proposed for sea-level rise modeling. This allows for more accurate predictions of which areas will be inundated, enabling proactive measures to protect critical infrastructure, agricultural plains, and densely populated low-lying areas.
- Erosion Monitoring and Mitigation: This data is indispensable for designing and implementing effective coastal defense strategies, from soft solutions like dune restoration to the planning of hard infrastructure where necessary.

To bridge the gap between the theoretical, scale-dependent nature of the Lebanese coastline and the practical necessity for fixed-length values in scientific and policy applications, we have established a suite of standardized, fit-for-purpose coastline lengths. This methodology links the characteristic scale of observation for a given discipline—such as coastal geomorphology or international law—to a theoretically justified measurement unit, or "stick length" (ε). This theoretical unit is then matched to the closest empirically measured stick length (s) from our Divider Stick analysis, ensuring that the final value is grounded in actual, robust data. Furthermore, each selected data point has been validated for its statistical reliability, falling well within the high-confidence regime of our measurements (see table 1). Table 4 provides a comprehensive set of defensible coastline lengths for a range of geographic and geologic applications, thereby resolving the "Coastline Paradox" for practical use.

Table 4. Standardized Lebanese Coastline Lengths by Discipline (see also Cocquempot *et al.*, 2019; Mclaughlin, & Cooper, 2010)

Discipline	Primary Purpose	Typical Scale of Observation	Theoretically Justified Stick Length (ε)	Discipline-Specific Maximum Stick Length (s_max)	Resultant Minimum Valid Coastline Length L(s_max)
Coastal Geomorphology	Studying micro-features of erosion & deposition (notches, sea caves, beach cusps)	> 1:1,000	10m	10m	381.3km

Sea-Level Rise Modeling	Predicting shoreline inundation; requires highest vertical & horizontal accuracy	1:5,000	20m	20m	350.6km
Coastal Zone Management	Local planning, engineering, habitat mapping	1:5,000	50m	40m	322.3km
Topographic Mapping (Standard)	General-purpose land navigation, infrastructure planning (e.g., national survey maps)	1:50,000	150m	160m	272.5km
National Resource Accounting	Standardized valuation of coastal assets for economic and ecological reporting	1:25,000	250m	320m	247.9km
International Law	Defining legal baselines for territorial waters, ensuring diplomatic and legal stability	1:100,000	500m	640m	231.5km
Regional Geologic Mapping	Mapping major rock formations and structural faults intersecting the coast	1:250,000	1,000m	1,280m	222.3km
Cartography (National Atlas)	General geographic representation for overview and educational clarity	1:1,000,000	2,000m	2,560m	215.6km

While this study provides a robust analysis of the Lebanese coastline's fractal dimension at a specific point in time, it is acknowledged that coastlines are dynamic systems subject to temporal variations. Although major geological features change over much longer timescales, seasonal and annual fluctuations driven by factors such as sediment transport, storm events, and anthropogenic modifications can alter the coastline's morphology at finer scales. Conducting a multi-temporal analysis on a series of satellite images captured over several decades would quantify the rate and nature of coastline changes.

5. Conclusion

This study estimated the fractal dimension of the Lebanese coastline using two complementary methodologies: the box-counting method and the divider (stick) method, applied to a coastline derived from Sentinel-2 satellite imagery. After careful inclusion of major anthropogenic coastal structures, the fractal dimension (*D*) of the Lebanese coastline was determined to be approximately 1.0805 (box-counting, $R^2 = 0.999$) and 1.0852 (divider method, $R^2 = 0.999$). These closely agreeing values indicate that the Lebanese coastline possesses a fractal nature, exhibiting statistical self-similarity over a wide range of scales. Its complexity is slightly greater than a simple Euclidean line but places it at the lower end of the spectrum compared to more highly convoluted coastlines globally. A key implication of this fractal character is that there is no single, absolute "true" length of the Lebanese coastline; length is a scale-dependent property. The highest resolution length measured is 381.270km, obtained with a 10 meter stick, representing the most appropriate value to cite for the coastline's extent incorporating the finest details captured. The fractal dimension $D = 1.0852$, as determined by the divider method for stick lengths (*s*) from 10m to 20480m, stands as the most robust, scale-independent characterization of the coastline's geometric complexity over the range where fractal scaling is observed.

While the fractal dimension is the primary scientific characterization, this study's principal practical contribution is the establishment of a framework for deriving a suite of standardized, 'fit-for-purpose' coastline lengths. This directly addresses the persistent challenge faced by policymakers, cartographers, and managers who require a single, defensible value for applications ranging from legal demarcation to national resource accounting. By linking the scale of observation to an empirically validated measurement, we resolve the "no true length" paradox into a set of functional, context-specific values.

Consequently, this research proposes that national bodies move beyond citing a single, ambiguous coastline length. We recommend the formal adoption of the multi-value framework, which provides a full spectrum of standardized lengths. These range from a high-resolution value of 381.3km for detailed coastal geomorphology down to a generalized value of 215.6km for national-scale cartography, with specific, defensible values provided for legal, economic, and planning purposes in between. This approach provides the necessary scientific justification for these standards, ensuring clarity, consistency, and defensibility for all stakeholders.

This study resolved the apparent paradox between Lebanon's complex tectonic setting and its coastline's low fractal dimension, where the dominant tectonic elements—such as the Mount Lebanon range and major fault systems—run parallel or sub-parallel to the coast. This alignment apparently limits the development of large-scale coastal irregularities. This structural control is consistent with findings from the Turkish Black Sea coast. In contrast, the highly indented Aegean and Mediterranean shores of Turkey have a much higher fractal dimension, reflecting a tectonic regime dominated by crustal extension from the rollback of the African slab. Therefore, the geometry, orientation, and kinematics of major structures dictate the fractal characteristics of neo-tectonically active coastlines. The findings contribute to a better understanding of Lebanon's coastal geomorphology and provide a valuable baseline for future research, including:

- Monitoring coastal changes and erosion/accretion patterns over time.
- Investigating variations in fractal dimension along different geological or geomorphological segments of the coastline.
- Assessing the impact of sea-level rise on coastal complexity.
- Comparing the fractal dimension obtained using different data sources (*e.g.,* higher-resolution imagery, historical maps) or further refinements of the methods.

Furthermore, the developed methodological framework, employing both box-counting and divider techniques, offers a replicable approach for analyzing the fractal characteristics of other coastlines, particularly where distinguishing or explicitly including anthropogenic influences is important for coastal zone management.

Future research could focus on leveraging machine learning (ML) and deep learning (DL). While the NDVI-based approach is robust, it relied on selecting a threshold and it required manual editing to correct for complex features, introducing potential for operator-induced bias. Traditional machine learning algorithms, such as Support Vector Machines (SVM), can be trained on multispectral data, using the spectral signatures across multiple bands to classify each pixel as either land or water, and distinguish between turbid water, submerged vegetation, and wet sand. Deep learning, particularly using Convolutional Neural Networks (CNNs), uses an architecture designed for semantic segmentation (such as U-Net or DeepLab), a model can be trained to perform pixel-level classification on an entire satellite image.

This is the first multi-method fractal analysis of the Lebanese coastline, with a scale-independent measure of its geometric complexity ($D \approx 1.085$), linking it to regional tectonic and sedimentation controls. For practical applications, it translates the theoretical 'coastline paradox' into a functional policy tool by creating a suite of standardized, 'fit-for-purpose' coastline lengths. This research equips policymakers, coastal managers, and legal experts with an empirically grounded framework.

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